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# Engineering Notes

## Flutter of an Aircraft Flying on Mars

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### Nomenclature

| | | |
|---|---|---|
| $a$ | = | axis location |
| $b$ | = | half of chord length |
| $C_L$ | = | lift coefficient |
| $c$ | = | sound speed |
| $g$ | = | gravitational acceleration |
| $I_\alpha$ | = | mass moment of inertia per unit span about axis $x = ba$ |
| $K_h$, $K_\alpha$ | = | spring constants |
| $M_{\text{tot}}$ | = | total mass of an airplane |
| $m$ | = | wing mass per unit span |
| $m/\pi\rho b^2$ | = | density ratio |
| $r_\alpha$ | = | dimensionless radius of gyration, $\sqrt{I_\alpha/mb^2}$ |
| $S$ | = | wing area |
| $U$ | = | flight velocity |
| $U_D$ | = | torsional divergence speed |
| $U_F$ | = | flutter speed |
| $x_\alpha$ | = | dimensionless static unbalance |
| $\rho$ | = | atmospheric density |
| $\omega_h$ | = | uncoupled natural bending frequency, $\sqrt{K_h/m}$ |
| $\omega_h/\omega_\alpha$ | = | bending-torsion frequency ratio |
| $\omega_\alpha$ | = | uncoupled natural bending frequency, $\sqrt{K_\alpha/I_\alpha}$ |

*Subscripts*

| | | |
|---|---|---|
| $e$ | = | Earth |
| $m$ | = | Mars |

### Introduction

RECENTLY, a Mars airplane was proposed that can explore a wider range than the Rover type explorer and with a higher image dissection than a satellite-type explorer. The atmospheric density, sound speed, and gravitational acceleration on Mars are different from those on Earth (Table 1). These differences cause differences in the design of an airplane between Mars and Earth. In this Note, we compared the wing flutter speeds on Mars and Earth. We considered the identical airplane flying on Mars and Earth and assumed the following.

1) The lift coefficient is the same on Earth and Mars.
2) The effect of compressibility is neglected.

The equilibrium in the vertical direction is given by

$$M_{\text{tot}}g = 0.5\rho U^2 SC_L \tag{1}$$

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The forward velocity is given by

$$U = \sqrt{2M_{\text{tot}}g/\rho SC_L} \tag{2}$$

According to Table 1 and Eq. (2), the ratio of $U$ on Mars to that on Earth is 6.2. In the present discussion, the $U_m/U_e$ is 6.2.

### Numerical Analysis Result

We analyzed the bend twist ($h-\alpha$) flutter of a two-dimensional wing in the incompressible flow shown in Fig. 1. The calculation process is indicated in [2]. Figure 2 shows the relations between the dimensionless flutter speed $U_F/b\omega_\alpha$ and the vibration frequency ratio $\omega_h/\omega_\alpha$. In these figures, $r_\alpha^2 = 1/4$ and $a = -0.4$. The parameters in these figures are $x_\alpha$ and $m/\pi\rho b^2$. The values of these parameters are shown in the figure captions with the dimensionless torsional divergence speed:

$$\frac{U_D}{b\omega_\alpha} = \sqrt{\frac{m}{\pi\rho b^2}\,\frac{r_\alpha^2}{(1+a)}}$$

The $U_{Dm}/U_{De}$ is 10, because the ratio of atmospheric density of Mars to that of Earth is about 0.01, as shown in Table 1. This $U_{Dm}/U_{De}$ is larger than the $U_m/U_e$, which is 6.2. Then torsional divergence on Mars does not occur on the wing of an airplane where torsional divergence does not occur on Earth.

Figures similar to Fig. 2 are shown in [2], where figures $m/\pi\rho b^2 = 2$–20. Considering the atmospheric density ratio between Mars and Earth in Table 1, we calculated the flutter speed for such density ratios as $m/\pi\rho b^2 = 10$, 100, 500, and 1000. The flutter speeds for $m/\pi\rho b^2 = 10$, which were obtained in the present analysis, are shown by solid lines in Fig. 2a. The values shown in [2] are also shown by dashed lines in this figure. This figure shows that the values for $m/\pi\rho b^2 = 10$ nearly agree those in [2]. Figure 2 shows that flutter speeds tend to increase when the density ratio increases.

**Table 1 Comparison of physical properties on Mars [1] and Earth**

| Physical properties | Earth | Mars |
|---|---|---|
| Atmospheric density $\rho$, kg/m$^3$ | 1.23 | 0.0118 |
| Sound speed $c$, m/s | 340 | 220 |
| Gravitational acceleration $g$, m/s$^2$ | 9.8 | 3.7 |

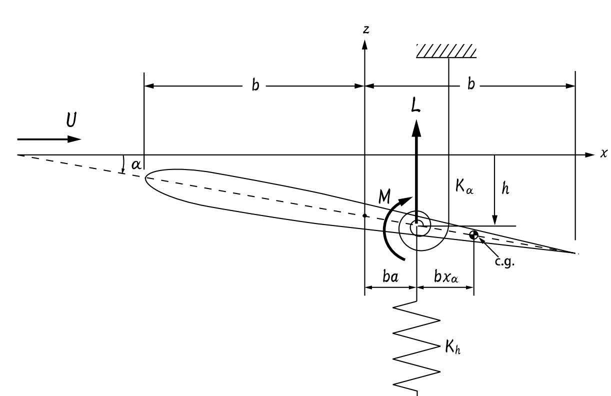


**Fig. 1 Wing of unit span in two-dimensional flow [2].**

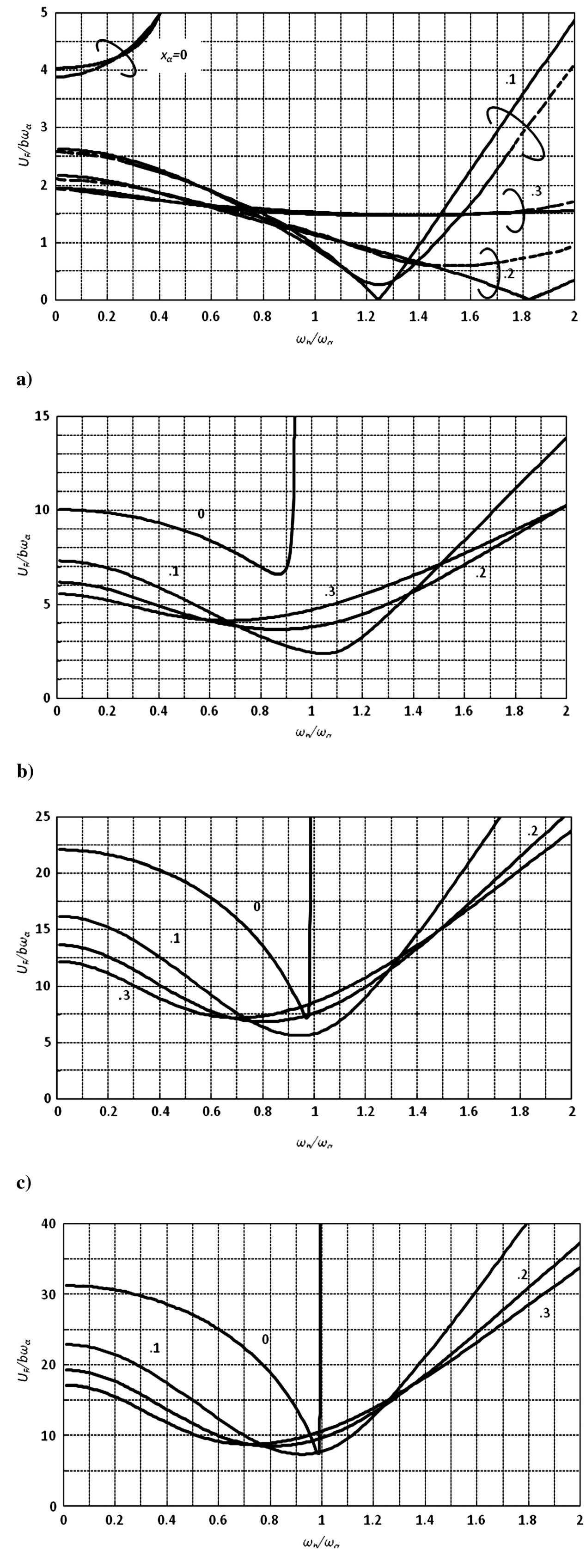


**Fig. 2 Dimensionless flutter speed $U_F/b\omega_\alpha$ plotted against frequency ratio $\omega_h/\omega_\alpha$ for various values of dimensionless static unbalance $x_\alpha = 0$, 0.1, 0.2, and 0.3 ($a = -0.4$ and $r_\alpha^2 = 1/4$ ): a) $m/\pi\rho b^2 = 10$ and $U_D/b\omega_\alpha = 3.54$ (dashed lines show the values in [2]), b) $m/\pi\rho b^2 = 100$ and $U_D/b\omega_\alpha = 11.2$, c) $m/\pi\rho b^2 = 500$ and $U_D/b\omega_\alpha = 25$, and d) $m/\pi\rho b^2 = 1000$ and $U_D/b\omega_\alpha = 35.4$.**

This is also pointed out in [2], which means that the flutter speed on Mars is larger than that on Earth.

Figure 3 shows the ratio between $U_{Fm}$ and $U_{Fe}$. The lateral axis shows the values of $m/\pi\rho_e b^2$. The $m/\pi\rho_m b^2$ was approximately set $100m/\pi\rho_e b^2$ when $U_{Fe}$ was calculated. The result for $x_\alpha = 0$ in Fig. 3b is not shown, because the curves start at $m/\pi\rho_e b^2 = 27.6$. In Fig. 3c, the results for $x_\alpha = 0$ are not shown, because the $U_{Fm}$ and $U_{Fe}$ are infinity. In all the figures, the $U_{Fe}$ is infinity when the $m/\pi\rho_e b^2$ is less than that for $U_{Fm}/U_{Fe} = 0$. For such cases, flutter does not occur on Earth.

As stated above, the flight speed on Mars is 6.2 times as large as that on Earth. The lines of $U_{Fm}/U_{Fe} = 6.2$ are shown in these figures. When the $U_{Fm}/U_{Fe}$ is larger than 6.2, flutter on Mars does not occur on the wing of an airplane where flutter does not occur on

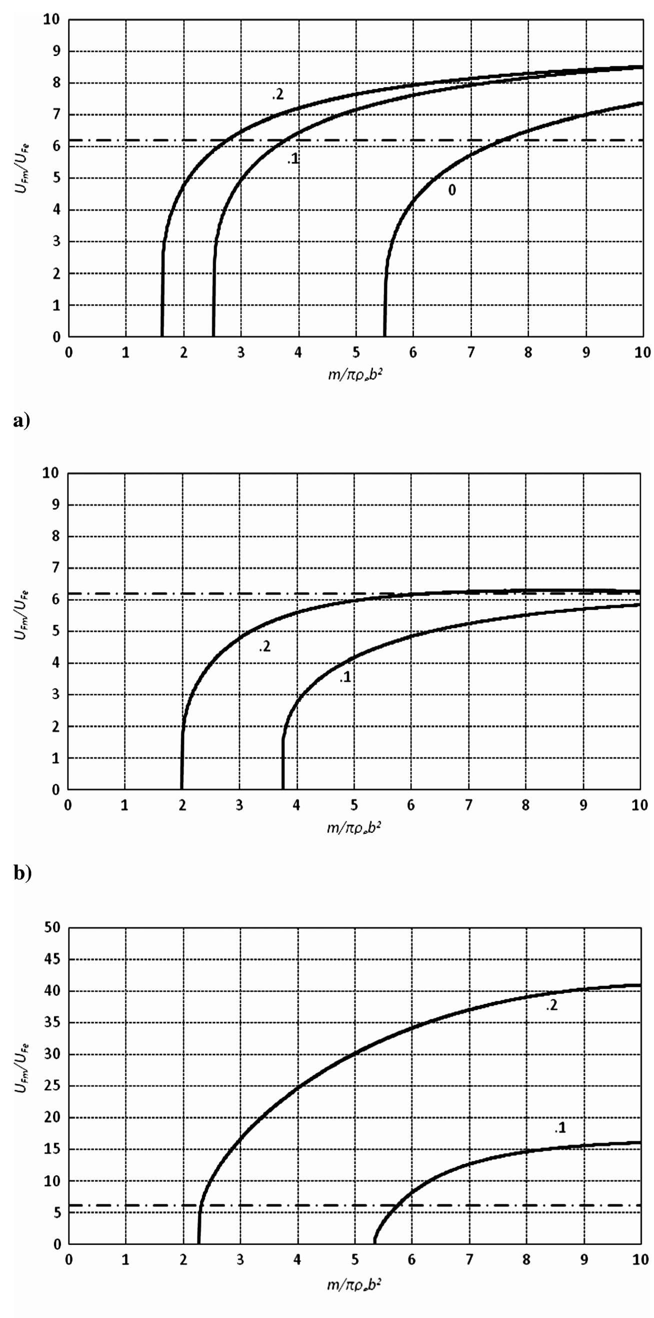


**Fig. 3 Ratio of flutter speed on Mars to that on Earth ($U_{Fm}/U_{Fe}$) plotted against density ratio on Earth ($m/\pi\rho_e b^2$) for various values of dimensionless static unbalance $x_\alpha = 0$, 0.1, and 0.2 ($a = -0.4$ and $r_\alpha^2 = 1/4$): a) $\omega_h/\omega_\alpha = 0.2$, b) $\omega_h/\omega_\alpha = 0.8$ (curve for $x_\alpha = 0$ starts at $m/\pi\rho_e b^2 = 27.6$), c) $\omega_h/\omega_\alpha = 1.5$ [for $x_\alpha = 0$ stable (that is, $U_{Fe} = U_{Fm} = \infty$) on both Earth and Mars].**

Earth. Conversely, when the $U_{Fm}/U_{Fe}$ is less than 6.2, flutter on Mars occurs on the wing of an airplane where flutter occurs on Earth. All the curves in Fig. 3 show that $U_{Fm}/U_{Fe}$ increases as $m/\pi\rho_e b^2$ increases. Therefore, a heavier wing is adequate for making $U_{Fm}/U_{Fe}$ larger than 6.2 and rejecting the possibility of flutter on Mars from a flight test on Earth.

## Conclusions

1) Flutter speeds on Mars were calculated, and the results are shown in figures as parameters of density ratio $m/\pi\rho b^2$, bending-torsion frequency ratio $\omega_h/\omega_\alpha$, and dimensionless static unbalance $x_\alpha$.

2) The possibility of wing flutter on Mars is compared with that on Earth. When a wing is lightened, we cannot eliminate the possibility of flutter on Mars from a flight test on Earth.

## References

[1] “Models of Mars’ Atmosphere (1974),” NASA SP-8010, 1974.
[2] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley Series in Mechanics, Addison-Wesley, Cambridge, MA1955.